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TECHNICAL MEMORANDUM

title: PAYBACK PERIOD ESTIMATOR FOR ENERGY SAVINGS, by

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ENCLOSURE (1)

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Technical Memorandum

PAYBACK PERIOD ESTIMATOR FOR ENERGY SAVINGS

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by
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SYNOPSIS

A formula, for computing payback times required for various energy-saving concepts requiring capital outlay, is derived and explained. Required formula inputs are initial investment, lost costs, salvage value of equipment to be replaced, interest rate, inflation rate, depreciation rate, and daily operation and maintenance costs for concept and for present modus operandi.

A numerical example using the formula is included. Finally there is discussion of the uncertainty in computed period, stemming from vagaries in predicted inputs.

BACKGROUND

There are many suggested new ways for supplying energy needs and improving efficiency of energy use. To implement each suggestion requires a different amount of capital outlay. Before actually selecting any particular idea for implementation it is desirable to have an estimate of the payback period.

INTRODUCTION

The payback period for a new idea is the length of time it must be in operation before the savings accumulated are sufficient to recover financially to the same point as if the present method of operation were continued. Thus, after the payback period the new method of operation becomes profitable.

Generally managers are reluctant to initiate a new modus operandi where the payback period is greater than 10 years and ideas promising less than 5 years to pay off are important candidates for implementation.

It is the purpose of this memorandum to set down a procedure for estimating payback period. Factors which complicate this are inflation rate, interest rate and depreciation (wear-out) rate. Here, depreciation is a physical degradation of equipment in contradistinction to an accountant's allowance for tax purposes.

MATHEMATICAL FORMULAS

It is convenient to compute payback period D' in units of days to avoid serious year-end mathematical discontinuities. The payback period Y' in units of years may then be computed as

$$Y' = \frac{D'}{365}$$

Define

d   daily depreciation rate compounded daily

d_y yearly depreciation rate compounded yearly

v daily interest rate compounded daily

v_y yearly interest rate compounded yearly

f daily inflation rate compounded daily

f_y yearly inflation rate compounded yearly

Inspection will verify the following correspondences:

$$d_y = 1 - (1 - d)^{365}$$

$$d = 1 - (1 - d_y)^{1/365}$$

$$v_y = (1 + v)^{365} - 1$$

$$v = (1 + v_y)^{1/365} - 1$$

$$f_y = (1 + f)^{365} - 1$$

$$f = (1 + f_y)^{1/365} - 1$$

Define

Y, D time in years or days respectively after new method is put to use

I price of new depreciable equipment at time zero required to initiate new method. The units are dollars at time zero

L costs of installing new equipment, a dollar loss at time zero. It is hoped the savings due to new method will save back this lost cost in Y' years

S salvage value of old equipment in time zero dollars. This is equipment to be replaced by new method

m, m_0 operation and maintenance costs (time-zero dollars) per day for the new and old methods respectively. It is hoped the daily savings $(m_0 - m)$ corrected back to current dollar units will accumulate to more than initial lost costs and cumulated depreciation losses and thus show profit after Y' years.

To assist in deriving an equation in the unknown D', which will balance net cost of the new proposed method with the existing method at time D', a preliminary computation will be made to find the total operation and maintenance costs in units of today's dollars for general unprimed D days where unused dollars are allowed to draw interest until needed to pay daily operating and maintenance bills. Recall that if today's operation and maintenance costs are m dollars, tomorrow's cost will be $m(1 + f)$, and the next day $m(1 + f)^2$, because of inflation. Also recall that a dollar invested today will be $1 + v$ dollars tomorrow and $(1 + v)^2$ the next, etc., because of interest accrued.

Thus the total operating and maintenance costs in today's dollars for D days is

$$m \ + \ m\frac{1 + f}{1 + v} \ + \ m\left(\frac{1 + f}{1 + v}\right)^2 \ + \ \dots \ + \ m\left(\frac{1 + f}{1 + v}\right)^{D-1} \ = \ m \sum_{k \, 1}^{D}\left(\frac{1 + f}{1 + v}\right)^{k-1}$$

$$= \ m \ \frac{1 - \left(\dfrac{1 + f}{1 + v}\right)^{D}}{1 - \dfrac{1 + f}{1 + v}}$$

This is for the new idea. For the present mode of operation merely change m to m_0.

Now recalling that the intrinsic (interest and inflation free) value of equipment tomorrow is $(1 - d)$ times that of today because of wear and aging, the balancing equation after D' days may be written in units of current dollars:

$$(m_0 - m) \ \frac{\left(\dfrac{1 + f}{1 + v}\right)^{D'} - 1}{\dfrac{1 + f}{1 + v} - 1} \ = \ I + L - S - (I - S) \ (1 - d) \left[\frac{(1 - d) \ (1 + f)}{(1 + v)}\right]^{D'-1} \tag{1}$$

An explicit solution of this equation for D' as a function of d, v, f, m_0, m, I, L and S is not known. However, iterative solution is rapid and a computer program for such solution is now operable. It is noted that the profit at the end of D days is

$$(m_o - m) \cdot \frac{\left(\dfrac{1+f}{1+v}\right)^{D} - 1}{\dfrac{1+f}{1+v} - 1} \;+\; (1 - S)(1 - d)\left[\frac{(1 - d)(1 + f)}{(1 + v)}\right]^{D-1} \;-\; (1 - S) - L$$

in units of time-zero dollars, paraphrased, O/M savings plus undepreciated net value minus (1 – S) minus L. When profit equals zero, $D' = D$. The computer program prints values for these components each half year.

A NUMERICAL EXAMPLE

This arbitrary example has been constructed simply for instructional purpose.

The swimming pool is presently heated with gas with furnace maintenance and fuel costs being three dollars per day now. Suggested is the discontinuance of this method of heating by the installation of a $7,000 mirror and magnifying glass solar energy transmission unit. It would require $1,500 lost cost to install this device and remove the gas heater which currently has a salvage value of $200. At today's labor rates it will cost $1 per day to clean and polish the mirror and magnifier and supply electricity to the reflector director. Projected inflation, depreciation and interest rates are respectively 0.06, 0.04 and 0.09 per annum. Find the economic payback period D'.

The parametric values are listed:

I	7,000
L	1,500
S	200
m	1
m_o	3
d_y	0.04
f_y	0.06
v_y	0.09

Using the program, Y' is computed to be 5.231342 years, i.e., D' equals 1,909.4398 days. The reader may wish to verify by use of a desk calculator that this does balance equation (1) at $3,553.32 while noting

$$d = 0.0001118348$$

$$f = 0.000159653$$

$$\nu = 0.000236131$$

The computer output printing is shown on page 13 of the Appendix.

STOCHASTICS

It is noted that inputs to this program include f_y, ν_y and d_y; rates which are not known precisely say some 5 years in advance. It may be argued that because it is intended to use the equation merely to compare an array of competing possible energy saving suggestions this imprecision in rate knowledge is not critical.

For those unconvinced by this argument, it is possible to place roughly some confidence bounds on computed Y' stemming from uncertainty in knowledge of future f_y, ν_y and d_y.

For example using the example of the last section, further assume due to the vagaries of the money market and maintenance procedures that 0.04, 0.06 and 0.09 are merely best estimates of the future values of d_y, f_y, ν_y respectively and that the standard deviation of these independent estimates are respectively 0.005, 0.01, and 0.01. How does this uncertainty of inputs translate to uncertainty in computed Y'?

To answer this, first, 6 additional computer runs were made holding I, L, S, m and m_o fixed, but varying d_y, f_y and ν_y systematically and independently. The following brief table displays results.

d_y	f_y	ν_y	Y'
0.040	0.06	0.09	5.231342
0.045	0.06	0.09	5.671017
0.035	0.06	0.09	4.823368
0.040	0.07	0.09	4.338149
0.040	0.05	0.09	6.487582
0.040	0.06	0.10	6.433594
0.040	0.06	0.08	4.352309

It is observed that a one standard deviation in d_y up and down translate to an average error in Y' of

$$\frac{(5.671 - 5.231) + (5.231 - 4.823)}{2} = 0.424 \text{ years,}$$

for f_y 1.075 years and for ν_y 1.041 years. These independent errors collectively amount to a standard deviation in Y' about equal to

$$\sqrt{0.424^2 + 1.075^2 + 1.041^2} = 1.555 \text{ years}$$

Assuming rate uncertainty to be normally distributed

$$1.96 \times 1.555 = 3.048$$

$$5.231 \pm 3.048 = 2.183, \ 8.279$$

That is to say the best estimate of payoff period is 5.2 years and, roughly, a 0.95 confidence interval for payoff period extends from 2.2 to 8.3 years.

It may be argued correctly that this interval is too wide because f and d are really not independent. Interest rates are notoriously highly correlated with inflation rates. More sophisticated methods are available for including such correlations in the computations.

DISCUSSION

Equation (1) is really quite versatile. For example many proposals are of such nature that once the new materials and equipment are installed they become useless for other purposes. Then I is simply zero and all of the initial expenses are L. Indeed when many people speak of payback period they mean time to pay off both I and L, neglecting any value the equipment may have after installation. In some cases the value of S to be used may actually be negative when it costs more to dismantle the present means of operation than salvage parts are worth.

One desiring to use this computer program may wish to prognosticate the mean effective rates for both inflation and interest. Financial analysts the last several years have in general failed to predict these even a year in advance very accurately. Pages 16, 17, 88, 89 and 90 of Reference 1 take the view that in most computations, ν_y should be set 0.10 above f_y in the belief that, for example, if inflation $f_y = 0.02$ actual interest (discount) rates will adjust to $\nu_y = 0.12$. Thus the user may ignore inflation by setting $f_y = 0$ and $\nu_y = 0.10$ because

$$\frac{1.12}{1.02} \approx \frac{1.10}{1.00}$$

The user will then be in agreement with NAVFAC P-442.[*] This philosophy asks for the real rate of return to be 10% per annum regardless of what level of future inflation may be in the offing, i.e.,

$$\frac{1 + \nu}{1 + f} = 1.1$$

To obtain a notion of how sensitive Equation 1 may be, relative to changes in input parametric values, the numerical example of this report was used to observe the change in Y' caused by deliberately changing some of the parameters one at a time,

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[*] Naval Facilities Engineering Command

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while holding the others fixed. Figures 1, 2, 3 and 4 display the results of this effort. It is observed that all four of these curves pass through Y' equal to 5.23 years at the abscissa value of the example. Figure 1 shows the change in Y' caused by changing the partitioning of (I + L) from all L to all I. Figure 2 shows the effect of changing the inflation rate alone and depicts what is known, i.e., the individual really believing double digit inflation will spend his money now while it still has value, and thus obtain a short payback period. Contrariwise, Figure 3 shows that if high interest rates are going to dominate, then one should not spend much money now, because money spent on capital goods will require a long payback period. Finally Figure 4 shows that for a given investment I, for a higher degradation rate, as expected, the payback period will be longer.

There are perhaps many slightly different rational techniques for estimating payback period, each with its own set of pros and cons. Equation (1) with its associated computer program may not necessarily be the best; however, it does join into one equation the main essential ingredients. It is most important when comparing two competing energy saving suggestions to use the same formula for computing payback period for each competitor.

Finally the joint impact of (a) less than 10 year payback, and (b) 10% real interest (above inflation); is a rather severe test. That is, any energy saving proposal passing the test should be implemented as soon as the money (I + L − S) can be made available. Fifty years ago when inflation was nil, 4% interest was a good rate.

RECOMMENDATION

That when using this program, careful thought be given to the numerical values of the inputs used at least in the following areas:

1. Note that time equals zero at the beginning of the day the competitive idea is to become actually operational, not some time earlier such as the inception of the idea, or the date of equipment purchase.

2. Partitioning initial costs (in units of time equals zero dollars) into I, L and S; being aware that in addition to installation cost, L may also properly include dismantling costs at a Y time of interest.

3. Be aware that all costs, returns and profits are reckoned in time zero dollars.

REFERENCE

1. Naval Facilities Engineering Command. NAVFAC P-442: Economic Analysis Handbook. Washington, DC, June 1975.

ACKNOWLEDGEMENTS

COMPUTER PROGRAM FOR
PAYBACK PERIOD ESTIMATION

```
      PROGRAM EMOP (INPUT,OUTPUT)
C     ENERGY OPTIMIZATION PROGRAM FOR PARKER-EATON.
      PRINT 107
      N=0
      DEL=.000001
      EPW=1./365.
    2 READ 100 ,XI,XL,S,XM,XMO,FY,RY,DY
      IF(XMO.LE.0.) STOP
      N=N+1
      PRINT 105,N
      PRINT 103,XI,XL,S,XM,XMO,FY,RY,DY
      DP=1.-(1.-DY)**EPW
      F=(1.+FY)**EPW-1.
      R=(1.+RY)**EPW-1.
      XMS=XI-S
      Y=(1.+F)/(1.+R)
      U=1.-DP
      YD=Y*U
      YDL=ALOG(YD)
      YL=ALOG(Y)
      YM1=Y-1.
      XMM=XMO-XM
      D=XMS/XMM
      DO 5J=1,30
      DM1=D-1.
      YTD=Y**D
      YD1=YD**(D-1.)
      DF=XMS*(YD1*U-1.)+(XMM*(YTD-1.))/YM1-XL
      ADF=ABS(DF)
      IF(DEL.GE.ADF) GO TO 10
      FPD=XMS*YD1*YDL*U+(YTD*XMM*YL)/YM1
      D=D-DF/FPD
    5 CONTINUE
      PRINT 101,  D,DF
C
C     IF CONVERGENCE ACHIEVED
C
   10 YR=D/365.
      PRINT 102
      PRINT 108,F,R,DP,D,YR
      FO=F
      RO=R
      DPO=DP
      T10=(XMM*(YTD-1.))/YM1
      T20=XMS*YD1*U
      PRINT 112
      PRINT 113
      PRINT 110
      DO=D
      YRO=YR
      DFO=DF
      D=0.
      DQ=S-XI-XL
```

```
      DUM=-XL
      PRINT 109,D,D,XL,XMS,XMS,DUM
      DO 15L=1,4
   11 D=D+182.5
      DM1=D-1.
      YTD=Y**D
      YD1=YD**(D-1.)
      T1=(XMM*(YTD-1.))/YM1
      T2=XMS*YD1*U
      DF=T1+T2-XMS-XL
      IF(DQ.GE.0..OR.DF.LE.0.)  GO TO 12
      PRINT 109,YR0,T10,XL,XMS,T20,DF0
   12 YR=D/365.
      PRINT 109,YR,T1,XL,XMS,T2,DF
      DQ=DF
      IF(DF.LT.0.) GO TO 11
   15 CONTINUE
      GO TO 2
  100 FORMAT(8F10.6)
  101 FORMAT(//4X34HNO CONVERGE 40 ITERATIONS, D DF = ,2F15.6//)
  102 FORMAT(1H0,9X1HF14X1HR14X1HD14X1HD14X1HY)
  103 FORMAT(1H0,8F15.6/)
  105 FORMAT(1H1,2X3HNO.,I3,1X1HI14X1HL14X1HS14X1HM13X2HMO15X2HFY13X2HRY
     *13X2HDY)
  107 FORMAT(1H1,20X27HENERGY DEPRECIATION PROGRAM)
  108 FORMAT(1H0,5F15.6///)
  109 FORMAT(1H0,6F15.3)
  110 FORMAT(1H , 9X5HYEARS 6X10HO/M SAVING11X1HL 9X5HI - S10X 9HNET VAL
     *UE 8X6HPROFIT)
     *ATION 6X6HPROFIT)
  112 FORMAT(1H0,22X4HPLUS11X5HMINUS 8X5HMINUS 9X4HPLUS)
  113 FORMAT(1H ,64X13HUNDEPRECIATED)
      END
```

DICTIONARY OF VARIABLES

Mathematical Symbol	Definition	FORTRAN Symbol
d	daily depreciation rate compounded daily	DP
d_y	yearly depreciation rate compounded yearly	DY
ν	daily interest rate compounded daily	R
ν_y	yearly interest rate compounded yearly	RY
f	daily inflation rate compounded daily	F
f_y	yearly inflation rate compounded yearly	FY
Y, D	time in years or days respectively after new method is put to use	YR, D
I	price of new depreciable equipment at time zero required to initiate new method. The units are dollars at time zero	XI
L	costs of installing new equipment, a dollar loss at time zero. It is hoped the savings due to new method will save back this lost cost in Y' years	XL
S	salvage value of old equipment in time zero dollars. This is equipment to be replaced by new method	S
m, m_o	operation and maintenance costs (time-zero dollars) per day for the new and old methods respectively. It is hoped the daily savings $(m_o - m)$ corrected back to current dollar units will accumulate to more than initial lost costs and cumulated depreciation losses and thus show profit after Y' years.	XM, XMO

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INPUT

7000.   1500.   200.   1.   3.   .06   .09   .04

```
000[][]00090000[]0000000[]]0000030000000000000000000000000[]G00000000]00000000000[]00000
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80
1111111111111[]1111111111111111111[]1111111111111111111111111111111111111111111111
2222222222222222222222[]22222222222222222222222222222222222222222222222222222222222
333333[]333333333[]33333333[]33333333[]33333333[][]333333333[]333333333[]33333333333[]333333
44444444444444444444444444444444444444444444444444444444444444444444444444444444[]4444
555555555555[]5555555555555555555555555555555555555555555555555555555555555555555555555
666666666666666666666666666666666666666666666666666666666[]6666666666666666666666666666
77[]7777777777777777777777777777777777777777777777777777777777777777777777777777777777
888888[]88888888[]8888888[]8888388[]8888888[]8888888[]8888888[]888888888[]88888888888[]888883
999999999999999999999999999999999999999999999999999999999999999999[]669999999999999999
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80
GLOBE NO.1 _____ STANDARD FORM 5021
```

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OUTPUT

NO. 15 I

	L	S	M	MO	FY	RY	DY
7000.000000	1500.000000	200.000000	1.000000	3.000000	.060000	.090000	.040000

F	R	D	D	Y	Y
.000160	.000236	.000112	.000160	5.231342	1900.039917

YEARS	PLUS O/M SAVING	MINUS L	MINUS I - S	PLUS UNDEPRECIATED NET VALUE	PROFIT
0.000	0.000	1500.000	6800.000	6800.000	-1500.000
.500	362.479	1500.000	6800.000	6570.787	-1366.734
1.000	719.935	1500.000	6800.000	6348.816	-1231.240
1.500	1072.437	1500.000	6800.000	6134.343	-1093.220
2.000	1420.055	1500.000	6800.000	5927.115	-952.830
2.500	1762.856	1500.000	6800.000	5726.887	-810.257
3.000	2100.906	1500.000	6800.000	5533.024	-665.670
3.500	2434.272	1500.000	6800.000	5346.496	-519.231
4.000	2763.018	1500.000	6800.000	5165.882	-371.100
4.500	3087.208	1500.000	6800.000	4991.371	-221.421
5.000	3406.906	1500.000	6800.000	4822.750	-70.340
5.231	3551.323	1500.000	6800.000	4746.677	.000
5.500	3722.174	1500.000	6800.000	4659.830	82.008
6.000	4033.073	1500.000	6800.000	4502.017	235.490
6.500	4339.668	1500.000	6800.000	4350.318	380.085
7.000	4642.006	1500.000	6800.000	4203.357	505.360

Figure 1.

Y' versus partition l + L = 8,500

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Figure 2.

Y' versus f_y

Y' versus ν_y

Figure 3.

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Figure 4.

Y' versus d_y

$|Y'|$

d_y